Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Linear Algebra-II

Subject Code : 4SC04MTC2 Branch : B.Sc.(Mathematics, Physics)

Semester : 4 Date: 18/04/2017 Time : 10:30 To 01:30 Marks : 70

Instructions:

1. (1) Use of Programmable calculator & any other electronic instrument is prohibited.
2. (2) Instructions written on main answer book are strictly to be obeyed.
3. (3) Draw neat diagrams and figures (if necessary) at right places.
4. (4) Assume suitable data if needed.

Q-1		**Attempt the following questions:**	**(14)**
	a)	Define: Distance between two vectors	(1)
	b)	Find angle between (-x , y ,w , z) and (y, x , z , w).	(1)
	c)	State any one property of determinant.	(1)
	d)	True/false: $(W^{\perp})^{\perp}$ = W .	(1)
	e)	Define : C[0,1].	(1)
	f)	Write the standard form of imaginary ellipse.	(1)
	g)	What is characteristic vector.	(1)
	h)	True/false: If T is symmetric linear transformation then the matrix associated with T is always symmetric.	(1)
	i)	What do you mean by Conics and Quadrics ?	(1)
	j)	Define : Orthonormal vectors.	(1)
	k)	Find inner product of (3,-1,6) and (5,-1,2) .	(1)
	l)	What is symmetric linear transformation.	(1)
	m)	True/false: Every orthonormal vectors are orthogonal.	(1)
	n)	If $T:R^3 \to R^4$ is orthogonal linear transformation and u = (1, 2, 3) then find $\|Tu\|$.	(1)

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	a)	Define : Orthonormal basis.	**(2)**
	b)	Find the angle between f and g where f(t) = t and g= h-3⟨h, f⟩f , $h(t) = t^2$, where $\langle f, g\rangle = \int_0^1 f(t)g(t)dt$.	**(4)**
	c)	State and prove Riesz-representation theorem .	**(8)**
Q-3		**Attempt all questions**	**(14)**
	a)	Prove that $W \cap W^{\perp} = \{0\}$.	**(2)**

b) Prove that $\|x\|=\|y\|$ iff $x - y \perp x + y$ **(4)**

c) Show that medians of triangle are concurrent. **(8)**

Q-4 **Attempt all questions** **(14)**

a) Define : P_v(u) **(2)**

b) Using gramschmidth orthogonalization process find orthonormal basis from the basis B = {(2,1,-1), (1,1,1),(0,1,2)}. **(4)**

c) Find P_v(u) and P_u(v) for the following. **(8)**

(1) u=(1,-1) , v=(-2,3) .

(2) u=$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, v = $\begin{bmatrix} 3 & 2 \\ 2 & -1 \end{bmatrix}$.

Q-5 **Attempt all questions** **(14)**

a) Define : Orthogonal linear transformation. **(2)**

b) If f : V→ V is any map such that **(6)**

I. f (0) = 0

II. $\| f(x)-f(y) \| = \| x-y \|$,

then show that f is orthogonal linear transformation.

c) With usual notation prove that **(6)**

$R_\theta=\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ and $\rho_\theta = \begin{bmatrix} cos\theta & sin\theta \\ sin\theta & -cos\theta \end{bmatrix}$

Q-6 **Attempt all questions** **(14)**

a) Show that $\det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}a_{22} - a_{12}a_{21}$. **(4)**

b) If A=$\begin{bmatrix} 0 & -1 & 3 \\ 2 & 5 & -4 \\ -3 & 7 & 1 \end{bmatrix}$, then find A^{-1} . **(6)**

a) Solve the system of equation by Cramer's rule x + y = 0 , y + z = 1, z + x = -1 . **(5)**

Q-7 **Attempt all questions** **(14)**

a) Find vol[v_1 , v_2 , v_3] where $v_1 = (\frac{-2}{3}, 0, 0)$, $v_2 = (0, \frac{5}{2}, 0)$, v_3=$(0, 0, \frac{-7}{5})$ **(2)**

b) If x=(x_1, x_2, x_3) and y = (y_1, y_2, y_3) then show that **(6)**

$$x \times y = \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix}.$$

c) $If \;\; A = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 2 & 0 & 8 & 0 \\ 3 & 6 & 9 & 0 \\ 4 & 7 & 10 & 1 \end{bmatrix}$ **(6)**

Then compute det A using column vectors and inner product.

Q-8 **Attempt all questions** **(14)**

a) Write only the standard equations for the following conics and quadrics. **(2)**

(1) Point Ellipse (2) Hyperbolic Cylinder.

b) Let T: V→V is symmetric linear transformation then show that the Eigen vectors v_1 and v_2 with respect to Eigen values λ_1 and λ_2 $(\lambda_1 \neq \lambda_2)$ are orthogonal. **(2)**

c) Reduce the equation 4xz+ $4y^2 + 8y + 8 = 0$ into standard form also determine the quadrics. **(10)**